Refraction Inversion
with
The GRM

An Introductory Tutorial
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1 Introduction

The data, objectives, references, etc.
The Mt Bulga Shear Zone

- The set of traveltime data to be examined in this tutorial was recorded in 1986 at Mt Bulga, which is located near Orange in southeastern Australia.
- It is the site of a narrow (5 - 10 m wide) massive sulphide ore body in steeply dipping to vertical Silurian meta-sediments.
- The site underwent extensive investigations several decades ago, but was subsequently abandoned as the ore body was considered to be uneconomic.
- The data were recorded on a relatively flat area away from the ore body and near the now-demolished core shed.
Objectives of the Tutorial

- To introduce the application of the GRM to near-surface seismic refraction methods.
- To demonstrate the application of simple spreadsheet methods for the evaluation of the inversion algorithms of the GRM.
- To address the issues of detailed spatial resolution.
- To introduce the concepts of non-uniqueness, aka, GIGO – garbage in, garbage out.
- To consider the implications for 3D refraction.
Palmer’s Mt Bulga References

Inversion algorithms, such as those of the GRM, generate starting models, which can be updated with inversion strategies, such as refraction tomography.
Defining vs Refining the Model

Starting Models
1. A starting model is always required with model based inversion.
2. Automatic inversion assumes a starting model, commonly with smooth vertical velocity gradients.
3. Alternate low resolution starting models are generated with the tau-p method.
4. Detailed starting models are generated with the GRM algorithms.

Starting Models are generated with Inversion Algorithms

Refraction Tomography

The Model
A starting model is always required. Define the model with parameters, such as velocities and depths.

The Response
Compare the response with the data. Compute updated model parameters. Iterate as required.

Refraction Tomography is an Inversion Strategy, more generally known as Model based Inversion

Compute the response

Update the model parameters (not the model!)
GRM vs Default Starting Models

- All methods of seismic inversion require a starting model, that is, a first “guess.”
- Traditional methods, most of which can be shown to be special cases of the GRM, employ inversion algorithms, which make various assumptions about the complexity of the model, which are discussed later in this course.
- This has usually been the “exact solution” or the “one shot in the locker” approach. It can also require professional “skills,” or experience, to be completely efficacious.
- By contrast, most default implementations of refraction tomography assume a “one size fits all” starting model.
- The smooth vertical velocity gradient model is a Claytons model - the starting model you have when you don’t have a starting model.
- The default model is then systematically updated until it is close to an “answer.” This is the ever decreasing circles approach. It usually does not require many professional “skills.”
- How many issues (financial, health, personal) in your life are you prepared to address with a simplistic “one size fits all” or a Claytons approach?
Claytons (From Wikipedia)

- Claytons is the brand name of a non-alcoholic, non-carbonated beverage coloured and packaged to resemble bottled whisky. It was the subject of a major marketing campaign in Australia and New Zealand in the 1970s and 1980s, promoting it as "the drink you have when you're not having a drink" at a time when alcohol was being targeted as a major factor in the road toll.

- Although the product is no longer being actively marketed, the name has entered into Australian and New Zealand vernacular where it represents a "poor substitute" or "an ineffective solution to a problem". It can also be used to describe something that is effectively in existence but does not take the appropriate name, eg. a common-law couple might be described as having a "Clayton's marriage".

- The description “Claytons” will be used for “smooth vertical velocity gradient” for conciseness.
Inversion Algorithms vs Inversion Strategies

- The inversion algorithms, such as those of the GRM, **DEFINE** the model, whereas tomography **REFINES** the model.
- In other words, inversion algorithms tell you what you don’t know, whereas tomographic inversion tells you what you already do know.
- If inversion algorithms are different from inversion strategies, why then is there an apparent conflict between GRM and tomography?
- Non-uniqueness! Or in the engineering ethos, risk and uncertainty!
- Many (most?) implementations of refraction tomography either ignore or reject the reality or the significance of non-uniqueness and uncertainty.
- Furthermore, the final tomogram is often very similar to the GRM starting model, where the traveltime data are accepted uncritically, indicating that tomography is not necessarily essential for achieving acceptable accuracy with the GRM.
Aleatory & Epistemic Uncertainty

- Precision refers to how closely individual measurements agree with each other. It is the misfit error aspect of tomography.
- Accuracy refers to how closely a measured value agrees with the correct value. It is the starting model aspect of tomography.
- Precision is a measure of exactness, whereas accuracy is a measure of rightness.
- The terms of precision and accuracy have largely been replaced with aleatory variability and epistemic uncertainty in most engineering treatments of risk.
- Refraction tomograms which are both accurate and precise are always preferable. Nevertheless, imprecisely accurate tomograms are invariably more useful than precisely inaccurate tomograms.
- In other words, minimizing epistemic uncertainty through the use of the most appropriate starting models is more important than minimizing aleatory variability or misfit errors.
Backus-Gilbert Appraisal

- In three landmark publications, Backus and Gilbert (1967, 1968, 1970), showed that linear inverse problems can have either a single unique solution or infinitely many solutions.
- In the latter case, it is possible to generate linear combinations of the data, which represent unique averages of the model.
- This process is known as appraisal, whereas model-based inversion, such as refraction tomography, is known as construction.
- The GRM inversion algorithms constitute a version of Backus-Gilbert appraisal, because they consist of simple linear combinations of the traveltime data. Few, if any, other geophysical methods share this unique and fortuitous coincidence.
- Accordingly, the use of the GRM can overcome a common source of non-uniqueness or epistemic uncertainty, namely the selection of the inversion algorithm used to compute the starting model (Oldenburg 1984).

The Ubiquity of Non-uniqueness

- The role of inversion is to provide information about the unknown numerical parameters which go into the model, *not to provide the model itself* (Menke, 1989, p2).


- In other words, tomography provides AN answer, which is directly related to the starting model, rather than THE answer.

- Non-uniqueness is a polite statement of GIGO – garbage in, garbage out!

- Non-uniqueness can be readily demonstrated with the somewhat questionable Claytons default starting models, as well as with a multiplicity of models generated with the GRM!
3 The Traveltime Data

The Mt Bulga consists of nine shot points. The data are reversed, that is, there are shot points at each end, in order to resolve any ambiguities between dip (aka structure) and seismic velocity.
Picking Traveltimes

- The adjacent picture is known as a “shot record”, or a field record.
- The shot point is located at station 1.
- The receivers are located from stations 26 to 73 at 5 m intervals.
- The shot record represents the amplified voltage from each receiver. The positive cycles are coloured black for clarity.
- The “first break traveltime” is usually measured where the “trace” kicks to the left.
- The traveltime at station 26 is ~75 ms.
- Palmer currently measures the traveltime and the amplitude at the first extremum, i.e., the first trough or sometimes, the peak.
- This traveltime is less sensitive to the signal-to-noise ratio of the data.
Task 1 – Presenting the Data

- You have been provided with a spreadsheet (with partially completed GRM computations!) of the traveltimes for the Mt Bulga shear zone.
- The spreadsheet is available upon request from d.palmer@unsw.edu.au
- Go to the “Data” worksheet.
- Your first task is to generate a graph of the traveltime data.
- It is essential that you correctly annotate the axes!
- Carefully consider the labeling of the horizontal axis. Station or CDP numbers are standard with seismic reflection methods. Distance is more common with geotechnical investigations.
- In the next slide, the labeling of the graphs corresponds with the location of the shot points. This method is often more convenient than file or shot numbers. It is also more convenient with single pass or CDP methods of data acquisition.
- It is desirable that you experiment with the symbols and colours to significantly improve the clarity of the graphs.
- Discuss why and when such presentations are important.
Mt Bulga Traveltime Data

- It is essential that you experiment with the presentation so that it represents your work.
- Examine the data for any inconsistencies.
Parameterizing the Traveltime Graphs

The first step in the processing of seismic refraction traveltimes is to assign each arrival to a refractor.
Ambiguity of Traveltime Graphs

There is a fundamental ambiguity between seismic velocity and dip or structure on a refracting interface, which is readily solved by recording data in both the updip and the downdip directions.

Another important ambiguity to be resolved is the number of layers detected.

The traveltime graphs show three segments for which the apparent seismic velocities are 1000 m/s, 2000 m/s and 5000 m/s in both the updip and downdip directions.

These reversed traveltime graphs are ambiguous, because they could represent either a 1D model with three layers bounded with horizontal plane interfaces or a 2D model with two layers separated by a synclinal interface.
Resolution of the Multilayer Ambiguity

- With the three layer earth, the traveltime graphs are similar, irrespective of the location of the source or the orientation of the geophone spread.
- Therefore, the traveltime graph for the shot at station 0 is essentially the same as that at station 12, except that it is offset or displaced to the left by twelve station intervals or 60 metres.
- In turn, the hinge points, the points where another refractor is detected, migrate or are displaced laterally.
- For example, the change from the 2000 m/s layer to the 5000 m/s layer at station 25 on SP12, has migrated laterally to station 13 for SP0.
Resolution of the Structural Ambiguity

Although the traveltime for SP12 shows three segments, it can also be produced by only two refractors. The first segment, shown with crosses, has a seismic velocity of 1000 m/s and represents the first layer. The second segment, shown with opaque circles, has an apparent velocity of 2000 m/s and represents a downdip velocity in the second layer, which has a true seismic velocity of 2820 m/s and a dip of 9.2 degrees. The third segment, also shown with opaque circles, has an apparent velocity of 5000 m/s and represents an updip velocity in the second layer. Therefore, the change in apparent velocities from the second segment to the third segment, is directly related to a simple change in dip of the same interface. The hinge point of the syncline occurs at a fixed position in the subsurface, in this case at station 24. Therefore, the change in slope of the traveltime graphs will always occur at the same receiver location in this case at station 25, irrespective of the offset of the additional shot points. An examination of the traveltime graphs shows that the hinge point associated with the change in dip, migrates vertically from SP12 to SP0.
Task 2 – Parameterize the Data

- The traveltime data show that three layers can be recognized.
- They are (i) a surface soil layer with a seismic velocity of 300 – 500 m/s, (ii) a weathered layer with a seismic velocity of ~1000 – 1200 m/s for which the upward concave graphs indicate cycle skipping, and in turn, the occurrence of a velocity reversal, and (iii) the sub-weathering with a seismic velocity of 2500 – 6000 m/s.
- Highlight the graphs accordingly.
- This task can be ambiguous and even challenging, because the apparent seismic velocities in the sub-weathering can be similar to those in the weathering.
The conventional reciprocal method (CRM) is probably the most commonly used and one of the most convenient methods for processing and interpreting near-surface seismic refraction data.
Origins of the Reciprocal Method

- The conventional reciprocal method (CRM) is probably the most commonly used and one of the most convenient methods for processing and interpreting near-surface seismic refraction data.

- The CRM had its origins in the 1930's, when it was known as the method of differences (Edge, and Laby, 1931; Heiland, 1963). In Japan the CRM is known as Hagiwara's method (Hagiwara, and Omote, 1939; Kitsunezaki, 1965), in Australia as the reciprocal method (Hawkins, 1961), in Europe, as the plus-minus method (Hagedorn, 1959), while in the USA it is known as the ABC method (Nettleton, 1940; Waters, 1981). All of these methods are mathematically identical.

- The advantage of using the CRM is that it is able to conveniently accommodate and resolve simple departures from plane interfaces and homogeneous velocities.

- In addition, most of the computations are relatively simple arithmetic operations, which can be conveniently carried out with spreadsheets.
References – Reciprocal Method

- Hagiwara, T, and Omote, S, 1939, Land creep at Mt Tyausu-Yama (Determination of slip plane by seismic prospecting): Tokyo University Earthquake Research Institute Bulletin, 17, 118-137.
Velocity Analysis Algorithm – 1

- The CRM refractor velocity analysis function $t_V$, is defined with the following algorithm:

$$ t_V = \frac{1}{2} \left( t_{FG} - t_{RG} + t_{FR} \right) $$

- This algorithm is applied to all pairs of reversed traveltimes at the receivers G, which are located between the forward shot point F, and the reverse shot point R, and which are from the same refractor.
When the traveltime expressions are substituted into the expression for the refractor velocity analysis algorithm, the result is:

\[ t_V = \sum_{j=1}^{n-1} Z_{jF} \frac{\sqrt{V_n^2 - V_j^2}}{V_n V_j} + FG \cos \theta / V_n \]

The spatial derivative or gradient of this function with respect to the shot to receiver distance FG, which is the reciprocal of the seismic velocity in the refractor, that is.

\[ \frac{d}{dx} t_V = \cos \theta / V_n \approx 1 / V_n \]
Task 3 – Computing $t_V$

- Go to the “CRM” worksheet.
- Compute and graph the refractor velocity analysis function $t_V$, for SP1 and SP97.

At station #26: $t_V(F26) = (C3-D3+F$52)/2$

- Use a reciprocal time, ie F$52, of 147 ms.
- How many changes in slope and therefore, changes in the seismic velocity, can you recognize? Make a note of the number.
- Estimate the seismic velocity between stations 51 and 61.
- Does the spatial derivative and therefore, the seismic velocity, change if the reciprocal time is altered to 157 ms?
The Velocity Analysis Function

- The graph presents the refractor velocity analysis function computed with the traveltimes for SP1 and SP97.
- The traditional method for computing seismic velocities has been to print out the graph, and to manually determine the reciprocal of the lines of best fit.
Task 4 – Computing the Velocity

- In this study, continuous seismic velocities can be determined with simple gradient methods.
- The velocity analysis function is averaged over the distance $\Delta x$, which is centred on the ends of the chord, in order to accommodate traveltime variations caused by the surface soil layers (which can exhibit very low seismic velocities) and for minor errors in picking the traveltimes.
- At station 27, for $\Delta x=10\text{m}$,
  
  $\text{@#27: } V(H4) = 5000 \times (A5-A3) / (\text{AVERAGE}(F4:F6) - \text{AVERAGE}(F2:F4))$

- At station 29, for $\Delta x=20\text{m}$,
  
  $\text{@#29: } V(I6) = 5000 \times (A8-A4) / (\text{AVERAGE}(F6:F10) - \text{AVERAGE}(F2:F6))$

- With the increase in the length of the chord $\Delta x$, there is an increase in stability, that is, a reduction in the number of spikes. However, it tends to reduce the resolution by smoothing, such as in the region between stations 51 and 61.
- What is the optimum trade off between resolution and stability? Compute seismic velocities for $10\text{ m} \leq \Delta x(10\text{ m}) \leq 50\text{ m}$. 
Seismic Velocities in the Sub-weathering

- Earlier, you were asked to inspect the velocity analysis function and determine the number of regions with distinct seismic velocities.
- Compare your earlier assessment with the results of the simple continuous gradient method.
Velocity Analysis – Mt Bulga

- A number of interfaces can be analysed with the careful selection of suitable pairs of shot records.

- Reversing the “forward” and “reverse” shot records does not change the spatial derivative (ie., the slope) nor therefore, the seismic velocity.

- For example, compare SP1 & SP97 with SP97 & SP1.
The time model function $t_G$, is defined by the following equation:

$$ t_G = \frac{1}{2} \left( t_{FG} + t_{RG} - t_{FR} \right) $$

This formula is applied to all pairs of reversed traveltimes at the receivers G, which are located between the shot points F and G, from the same refractor.
Task 5 – Computing $t_G$

- The time model at station 26 is:
  
  $$t_G(N3) = \frac{(C3+D3-N52)}{2}$$

- Again, you are required to QC your calculations with a simple graph.
Time Model – Mt Bulga

- Through the careful selection of suitable pairs of shots, a number of interfaces can be analysed.
- Note the close agreement between the time model and the HALF intercept times.
Computing the Depth Model

- The depth model is computed from the time model(s) using the seismic velocities.

\[ t_G = \sum_{j=1}^{n-1} Z_{jG} \cos i_{jn} / V_j = \sum_{j=1}^{n-1} Z_{jG} \sqrt{V_n^2 - V_j^2} / V_n V_j \]

- The formula is recursive, i.e., each layer thickness is computed sequentially, starting from the surface.

\[
Z_1 = t_{G-1} \frac{V_1_{\cos i_{12}}}{V_1} = t_{G-1} \frac{V_2 V_1}{\sqrt{V_2^2 - V_1^2}}
\]

\[
Z_2 = \left[ t_{G-2} - Z_1 \frac{\cos i_{13}}{V_1} \right] \frac{V_2}{V_1 \cos i_{23}} = \left[ t_{G-2} - Z_1 \frac{\sqrt{V_3^2 - V_1^2}}{V_3 V_1} \right] \frac{V_3 V_2}{\sqrt{V_3^2 - V_2^2}}
\]
Task 6 – Computing a Depth Model

- The weathered layer consists of TWO layers, viz a thin soil layer \( t_{G-1} = \sim5 \text{ ms} @ 400 \text{ m/s} \rightarrow Z_1 = \sim2 \text{ m} \) and a second layer with a seismic velocity of \( \sim1000 \text{ m/s} \).

- The time model is converted to a depth model by multiplication with the depth conversion factor, DCF, ie.,

\[
t_G = \sum_{j=1}^{n-1} Z_{jG} \cos i_{jn} / V_j = \sum_{j=1}^{n-1} Z_{jG} \sqrt{V_n^2 - V_j^2} / V_n V_j
\]

\[
DCF = \frac{V_n V_j}{\sqrt{V_n^2 - V_j^2}}
\]

\[
Z_1 = t_G \frac{V_2 V_1}{\sqrt{V_2^2 - V_1^2}} = t_G DCF
\]
Depth Cross Section – Mt Bulga

- The standard approach to depth computations is to calculate the thicknesses of each layer.
- Frequently however, the surface soil layers are not well defined with either the time model or the seismic velocities.
- Accordingly, an average seismic velocity is often used for convenience.
- In such cases, a posteriori information, such as borehole control, is often useful to fix depths more accurately.
Refraction Tomography

- The CRM average depth model has been converted into an XYZ file with MATLAB and then gridded with SURFER.
- Note the gridding artifacts.
- It is shown as a “starting model” for refraction tomography.
- The starting model is then “cosmetically” refined with refraction tomography so that it agrees more closely with all of the traveltime data.
- Note that there are very few significant visual differences between the two images, even though the misfit errors of the starting model (5.53 ms) are larger than those of the final WET tomogram (1.24 ms).
Reciprocal Method – A Summary

- The reciprocal method (CRM) is one of the most commonly used methods for processing shallow seismic refraction data. In the USA, it is known as the ABC method, in Europe as the plus-minus method and in Japan as Hagiwara’s method.
- The CRM algorithms employ forward and reverse traveltine data which are recorded at receivers located between two shot points and which are from the same refractor.
- The refractor velocity analysis algorithm is:
  \[
  t_V = \frac{1}{2} \left( t_{\text{forward}} - t_{\text{reverse}} + t_{\text{reciprocal}} \right)
  \]

  The most important feature of the refractor velocity analysis algorithm is the subtraction of the forward and reverse traveltimes, which essentially minimises the effects of any variations in the thicknesses of the upper layers.
- The reciprocal of the spatial derivative of the refractor velocity analysis function is the seismic velocity in the refractor.
- The velocity analysis algorithm of the CRM is able to determine large scale or long wavelength changes in refractor velocity. The CRM is unable to resolve the seismic velocities in regions which are narrow in relation to their depth.
- The refractor velocity analysis algorithm of the CRM can generate narrow zones of high and low seismic velocities in the refractor where there are rapid changes in depth to the refractor, but which are artefacts of the inversion algorithm. Any lateral changes in seismic velocities in the refractor which are associated with changes in depths, should be treated with considerable caution.
- The intercept of the refractor velocity analysis function at the shot point is the time-depth at the shot point.
- The time model algorithm is:
  \[
  t_G = \frac{1}{2} \left( t_{\text{forward}} + t_{\text{reverse}} - t_{\text{reciprocal}} \right)
  \]

  The addition of the forward and reverse traveltimes effectively converts the moveout or the component of the traveltine in the refractor to a constant value which is then removed with the subtraction of the reciprocal time.
- The time model algorithm can define irregular refractors. It can accommodate irregular topography through using the topography as a floating datum.
6
The GRM Algorithms

The GRM Refractor Velocity Algorithm

- The refractor velocity analysis algorithm is computed for a range of XY values, which are integer multiples of the station spacing.
- The value is referenced to G, which is midway between X and Y.
- For XY values which are EVEN multiples of the station spacing, G falls on an integer station number.
- For XY values which are ODD multiples of the station spacing, G falls midway between the integer station numbers.
- The refractor velocity function reduces to approximately the traveltime from the shot point at F to a point in the refractor below the reference point G.
- Therefore, the spatial derivative of the line(s) fitted to the points computed with this equation are a measure of the seismic velocity in the refractor.

\[
\frac{d}{dx} t_V = \frac{1}{V_{refractor}} = \frac{1}{V_n}
\]

\[
t_V = \frac{1}{2} \left( t_{FY} - t_{RX} + t_{FR} \right)
\]
The refractor velocity function has been evaluated for the traveltimes for SP1 and SP97, for \(-25 \text{ m} \leq XY(5 \text{ m}) \leq 30 \text{ m}\).

The traveltime graphs for shot points 1, 13, 85 and 97 represent arrivals from the base of the weathering, and since they are very irregular, they indicate that the base of the weathering is also very irregular.

In fact, the apparent seismic velocities are negative between stations 37 and 46 in the forward direction and between stations 47 and 49 in the reverse direction.

These very irregular traveltime graphs indicate that detailed inversion algorithms, which explicitly identify forward and reverse traveltimes, such as those of the GRM, are essential.

Default starting models are usually not able to distinguish apparent updip and downdip seismic velocities from true seismic velocities. As a result, artifacts are common.
Task 7 – Compute and Graph $t_V$

- Go to the “GRM” worksheet. Interpolate the travel times with simple averages.
- Compute the GRM refractor velocity analysis function for $-25 \text{ m} \leq XY(5 \text{ m}) \leq 30 \text{ m}$.
- Compute the AVERAGE.
- Construct a graph of the velocity analysis function.
- Determine the $XY$ value(s) for which the graph(s) are simplest. This will be your initial optimum $XY$ value(s).
- Compute the seismic velocity between stations 51 & 61. Is there any ambiguity in computing this value?
Refractor Velocity Analysis Function

- The graphs can become very confusing unless you carefully select the symbols and the colours.
- Note that the use of negative XY values ensure symmetry about the average.
- Although the average is an inevitable result of the range of XY values, nevertheless it still represents the simplest graph.
An objective approach to generating detailed seismic velocities is a novel application of the Hilbert transform.

It computes an average of the reciprocal of the seismic velocities derived over a range of chord lengths, such as 5, 10, 15, 20, 25, and 30 m which are multiples of the station spacing $\Delta x = 5$ m, and which are centred on the reference station:

At station #28.5,

$$V(V10) = \frac{6000}{((U11-U9)/5 + (U12-U8)/10 + (U13-U7)/15 + (U14-U6)/20 + (U15-U5)/25 + (U16-U4)/30)}$$
Task 8 – Computing Velocities

- Compute the seismic velocities for the averaged refractor velocity analysis function for the Mt Bulga shear zone.
- Use \( k = 6 \), i.e., use chord lengths of 5, 10, 15, 20, 25, and 30 m.
- Graph your results.
- Experiment with different values of \( k \).
- Compare the convenience and the detail achieved with the manual approach. Discuss reliability of both.
The GRM Time Model Algorithm

\[ t_G = \frac{1}{2} \left( t_{FY} + t_{RX} - \left( t_{FR} + \frac{XY}{V_n} \right) \right) \]

\[ \frac{XY}{V_n} = \overline{t_v(Y)} - \overline{t_v(X)} \]

- The time model algorithm \( t_G \), provides a measure of the depth to the refracting interface in units of time.
- The term \( \frac{XY}{V_n} \), represents additional traveltime in the refractor between the stations at \( X \) and \( Y \).
- This term is approximated with the difference in the refractor velocity analysis function, which has been averaged over a range of \( XY \) values.
Task 9 – Compute and Graph $t_G$

- Compute the GRM time model for $-25 \text{ m} \leq XY(5 \text{ m}) \leq 30 \text{ m}$.
- Compute the AVERAGE for $-10 \leq XY(5 \text{ m}) \leq 15 \text{ m}$.
- Construct a graph of the time model function.
- At station #25.5 for $XY = 5\text{ m}$,
  
  \[ t_G(AE4) = \frac{(F5 + G3 - AE$101 - (U5-U3))}{2} \]
7

Inhomogeneities in the Surface Soil Layer

The buried focus represents the inherent limit of resolution of all first arrival refraction data.

Any shorter wavelength irregularities originate in the surface soil layers where the very low seismic velocities generate relatively large traveltime anomalies.
Defining Surface Anomalies with GRM

- Surface soil traveltime anomalies are coincident, whereas those originating at the base of the weathering are offset by ~ optimum XY distance. Mt Bulga massive sulphide ore body traverse.
- With the time model computed with finite XY values, the surface anomalies in the forward and reverse traveltimes are separated by the XY distance and are half the value. However, there are relatively minor variations in the time model of the base of the weathering with various XY values.
- The AVERAGE of the time model smoothes the surface anomalies but preserves the time model of the base of the weathering.
- The surface time anomalies are obtained by subtracting the average from the time model for zero XY.
Correcting Surface Anomalies with GRM

- The corrections for surface inhomogeneities are usually approximately 1 ms.
- The corrections also compensate for minor picking errors.
The corrections for the inhomogeneities in the surface soil layers have the most significant impact on the refractor velocity analysis function.

The improved smoothness and symmetry about the optimum XY value improves the performance of velocity computations with the Hilbert transform, especially with averaging.

The effect is more significant with larger station intervals.
Near-Surface Inhomogeneities

- The very near-surface soil layers often exhibit very low seismic velocities, and as a result, any variations in either composition, moisture or thickness can result in variations in the travel times.

- These variations occur over quite short distances and correspond with the short wavelength static corrections for reflection data.

- The variations in the traveltimes caused by the near-surface inhomogeneities can adversely affect the determination of detailed seismic velocities in the sub-weathering.

- Corrections for the near-surface inhomogeneities can be computed with a novel application of the GRM time model algorithms.
Task 10 – Surface Statics Corrections

- Compute the traveltime corrections for surface soil layers by subtracting the AVERAGE time model from that computed with XY=0.
- Compute the corrected traveltimes by subtracting the corrections for the surface soil layers from the measured traveltimes.
Task 11 – Updated Computations

- Compute new traveltimes which are corrected for the near-surface soil inhomogeneities.
- Compute and graph the GRM refractor velocity analysis function and the time model with the updated traveltimes.
- The revised computations can be readily implemented with a simple copy and paste of the previous computations.
The application of the statics corrections to the traveltime data for the Mt Bulga shear zone improves the symmetry and smoothness of the graphs of the velocity analysis function.
The application of the statics corrections to the traveltime data for the Mt Bulga shear zone improves the smoothness of the graphs of the time model.
The averaging of the refractor velocity analysis function over a range of XY values can reduce the traveltime anomalies caused by variations in the surface soil layers.
Near-Surface Inhomogeneities

- The determination of the near-surface inhomogeneities with the averaging of the GRM time model over a range of XY values is reasonably effective.
- However, it is not completely effective. Although, it can be applied repeatedly, that strategy tends to over-smooth the traveltime data.
- The major effect of the near-surface inhomogeneities is to cause erratic or “noisy” seismic velocities where they are computed with the Hilbert transform.
- The seismic velocities are computed over distances as small as 5 m with the Mt Bulga shear zone. Since the time differences can be less than 1 ms for seismic velocities ~6000 m/s, variations as small as 0.1 ms can result in quite different values.
- An alternate approach employed in this short course is to average the refractor velocity analysis function over a number of XY values.
- The task is to average over a sufficient number of XY values, in order to achieve stable values, without over-smoothing the velocity model.
- Regularization with tomography can often provide further smoothing.
The computation of seismic velocities is an ill-posed problem.

Small variations in the averaged refractor velocity analysis function results in large variations in the seismic velocities.

The effective XY value is the average of all, or equivalently, the end XY values.

My preference? The range $-20 \text{ m} \leq \text{XY} \leq 25 \text{ m}$.
Task 12 – Optimum XY Window

- Average the refractor velocity analysis function over a range of XY windows.
- The suggested windows, which are symmetrical about XY = 2.5 m are:
  
  \[ 0 \leq XY \leq 5 \text{ m} \]
  \[ -5 \text{ m} \leq XY \leq 10 \text{ m} \]
  \[ -10 \text{ m} \leq XY \leq 15 \text{ m} \]
  \[ -15 \text{ m} \leq XY \leq 20 \text{ m} \]
  \[ -20 \text{ m} \leq XY \leq 25 \text{ m} \]
  \[ -25 \text{ m} \leq XY \leq 30 \text{ m} \]

- Compute the seismic velocities with the Hilbert transform and graph the results. Select the range of XY values which provide the best compromise between resolution and stability.
- Discuss your confidence in the lateral resolution of the seismic velocities with this method.
The Welcome Reef Shear Zone

- There are changes in depth and seismic velocity in the vicinity of station 48. They are representative of a major shear zone ~100 m wide.
- My optimum window of XY values over which the refractor velocity analysis function is averaged is ~13 XY values, ie. 
  \(-30 \text{ m} \leq XY(10) \leq 90 \text{ m}\)
The seismic velocities are computed between those stations where the refractor velocity analysis function is much the same for all XY values, viz., stations 28, 31, 37, 44, 50, 54, 61, 66, and 70.

These average interval seismic velocities, which are independent of the XY values, will be termed nodal seismic velocities.
In the vicinity of station 49 where the greatest variability occurs with the refractor velocity function, there is good correlation between the nodal velocities and the velocities derived from the averaging of ten XY values (-7.5 m ≤ XY(2.5 m) ≤ 15 m).

Although the compromise between resolution and stability is ultimately a decision for each individual practitioner, it has been the author’s preference to err on the side of resolution, largely because refraction tomography through the regularization process tends to smooth rather than to improve spatial resolution.
9

Optimum XY Value

The optimum XY value is readily derived with a minimal variance approach from suitable presentations of the seismic velocities in the subweathering.
The syncline model is quite simple. It has two layers with constant seismic velocities and a single change of dip in the interface.

- The downdip apparent seismic velocities are 2000 m/s while the updip values are 5000 m/s.
It is possible to compute seismic velocities in XY increments of half the station spacing, with the appropriate selection of the range of XY values over which the refractor velocity analysis function is averaged.

The optimum XY value is $15 \pm 2.5$ m.
Task 13 – Optimum XY Value

- Compute the averaged refractor velocity analysis function for the Mt Bulga data over the following ranges:
  - $-25 \text{ m} \leq XY \leq 20 \text{ m}$ (XY = -2.5 m)
  - $-20 \text{ m} \leq XY \leq 20 \text{ m}$ (XY = zero)
  - $-20 \text{ m} \leq XY \leq 25 \text{ m}$ (XY = 2.5 m)
  - $-15 \text{ m} \leq XY \leq 25 \text{ m}$ (XY = 5.0 m)
  - $-15 \text{ m} \leq XY \leq 30 \text{ m}$ (XY = 7.5 m)

- Compute the seismic velocities in the refractor with the Hilbert transform. Use chord lengths of 5, 10, 15, 20, 25 and 30 m ($5 \text{ m} \leq \Delta x(5 \text{ m}) \leq 30 \text{ m}$).
- Determine the optimum XY value.
Mt Bulga Shear Zone Velocities

- The optimum XY value for the Mt Bulga shear zone is $2.5 \pm 2.5$ m.
Corrections vs Resolution

- If the inhomogeneities in the surface soil layers are not addressed, then the use of detailed algorithms, e.g., the Hilbert transform, results in erratic values of the seismic velocities.

- In this study, effective corrections are achieved with a two stage process which, (i) removes the gross variations using the GRM time model, and (ii) averages the refractor velocity analysis function over a range of XY values.

- Have these processes resulted in any loss of resolution?
Mt Bulga Massive Sulphide Ore Body

- The XY value for which the seismic velocity model is the simplest with the least extreme values is taken as the optimum, which is $3.75\ m \pm 1.25\ m$. 
The Welcome Reef Shear Zone

- Two optimum XY values of 30 m and 60 m can be recognized.
10
Average Vertical Velocity

The GRM average vertical velocity is a unique feature of the GRM. It is analogous to the NMO velocity of seismic reflection methods.
The Optimum XY Value

- Model studies confirm that the optimum XY value corresponds with the XY value for which the forward and reverse ray paths emerge from the refractor at approximately the same point.
- Like the GRM time model, the optimum XY value is a function of the total depth and ALL of the seismic velocities. It increases with the seismic velocity in the weathering.
- In other words, there are NO issues with extrapolation from shallow penetration arrivals, or the ASSUMPTION of vertical velocity gradients.
- The horizontal layer approximation is adequate for dipping layers.
GRM Average Vertical Velocity

\[ \bar{V} = \sqrt{\frac{X_{Y_{\text{optimum}}}^2}{V_n^2} \left( X_{Y_{\text{optimum}}} + 2t_G V_n \right)} \]

- The GRM average vertical velocity is obtained simply by eliminating the total thickness term from the expressions for optimum XY value, and the GRM time model \( t_G \).
- The average vertical velocity in the weathering for the Mt Bulga shear zone is 500 m/s (average time model, \( t_G = 30 \) ms; refractor seismic velocity, \( V_n = 6000 \) m/s, optimum XY value = 2.5 m).
Average Vertical Velocity – Errors

\[ \frac{\Delta \bar{V}}{\bar{V}} = \frac{\Delta X Y}{2 X Y} \cos^2 \bar{i}, \text{ where } \sin \bar{i} = \frac{\bar{V}}{V_n} \]

\[ \frac{\Delta \bar{V}}{\bar{V}} \approx \frac{\Delta X Y}{2 X Y} \]

- The errors in the determination of the optimum X Y value are the major source of errors with the average vertical velocity.
- Generally, the error in the optimum X Y value is taken as half the station spacing.
- For the Mt Bulga shear zone, the error is 50%, that is, the average seismic velocity in the weathering is 500 ± 250 m/s.
- Despite the magnitude of the error, the average vertical velocity clearly indicates the occurrence of a velocity reversal, rather than constant seismic velocities (~1000 m/s in the traveltime graphs) or vertical velocity gradients.
Summary – GRM Average Velocity

- For most reasonable vertical velocity functions, there is very little penetration of the first arrivals in each layer.
- As a result, traditional seismic inversion methods generally *assume* constant seismic velocities.
- Alternatively, most implementations of refraction tomography *assume* smooth vertical velocity gradients.
- The optimum XY value is a function of ALL of the layers between the refractor and the surface.
- The GRM average vertical velocity is derived from the optimum XY value.
- The GRM average vertical velocity can accommodate constant seismic velocities, velocity reversals and seismic anisotropy.
11
Depth Computations

A number of depth models can be generated, depending upon the model of the seismic velocities, which have been either measured or assumed, in the weathering.
Task 14 – Depth Computations

- Compute the Depth Conversion Factor, DCF, where
  \[ DCF = \frac{V_1 \cdot V_2}{(V_2^2 - V_1^2)^{1/2}} \]

- Compute the depth where
  \[ \text{Depth} = \text{Time Model} \times DCF \]

- \( V_1 \) is the seismic velocity in the weathering – 1000 m/s, and \( V_2 \) is the seismic velocity in the sub-weathering.

- Repeat with \( V_1 = 500 \text{ m/s} \)

- Generate XYZ file in MATLAB, grid in SURFER and invert with RAYFRACT.
GRM vs Default Uncertainty

- The hyperbolic velocity function represents the limiting case of the maximum vertical velocity gradient, which is consistent with linear traveltime graphs.
- It provides the best measure of uncertainty with standard refraction methods, including tomography.
- Velocity gradients commonly overestimate depths by about 50%.

\[
\frac{\text{Depth}_{\text{hyperbolic velocity}}}{\text{Depth}_{\text{constant velocity}}} = \frac{2}{\pi} \sqrt{\frac{V_n + V_1}{V_n - V_1}} \cosh^{-1}\left(\frac{V_n}{V_1}\right)
\]

- The error in the GRM average vertical velocity is:

\[
\frac{\Delta \bar{V}}{\bar{V}} \approx \frac{\Delta XY}{2 XY}
\]
Which Depth Model(s)?

BEFORE you implement any drilling program, which depth model(s) should you ethically provide to the drilling engineers or the client?

How should you quantify uncertainty? See previous slide.
The spatial resolution of the seismic velocities is not improved with a detailed starting model, because the seismic velocities in the weathering, which uncritically accept the traveltime data, ignores the probability of a reversal in the seismic velocities.
The seismic velocities in the sub-weathering depend upon the XY value selected and the seismic velocity in the weathering.

The GRM average seismic velocity in the weathering is ~500 m/s, because of the probable occurrence of a reversal in the seismic velocity in the weathering.
Default starting models, such as the smooth vertical velocity gradient, the tau-p method and VIRT do not detect or define the low velocity shear zone, largely because it is not included any starting model.
Seismic refraction attributes are obtained by combining the seismic velocities with the head coefficient derived from the head wave amplitudes.

The P-wave modulus may eventually prove to be a more useful measure of rock strength.
The results of seven cross-lines from a later 3D survey are shown above. The 2D profile corresponds with inline 19, while the in-line station numbers are the same for both surveys.

The cross-line seismic velocities clearly confirm the existence of the 50 m wide zone with a low seismic velocity. Also, note the cross cutting faults.

Although borehole data is always useful, the considerably better spatial sampling of the 3D refraction results provide at least as compelling an interpretation of the low seismic velocities as a major shear zone as even a moderate number of boreholes.

Compute three models of the seismic velocities in the weathering, and three models of the seismic velocities in the subweathering, in order to explicitly address uncertainty.
GRM Inversion

- It is not unusual for GRM tomograms to be similar to the GRM starting models, irrespective of the XY value, where the traveltime data are accepted uncritically.
- Detailed models of the seismic velocities in the sub-weathering can be generated with a novel application of the Hilbert transform.
- The GRM can explicitly address epistemic uncertainty through computing a suit of tomograms, each of which satisfies the traveltime data to sufficient accuracy.
- Three models of the seismic velocities in the weathering should be computed, viz. (i) uniform seismic velocities computed with uncritical acceptance of the traveltime graphs, (ii) the hyperbolic equivalent of the uniform seismic velocities, and (iii) GRM average vertical velocities using the optimum XY value.
- Three models of the seismic velocities in the sub-weathering should be computed, using the GRM refractor velocity analysis for $XY = \text{optimum } XY \pm n \Delta x/2$, where $\Delta x = \text{station spacing and } 1 \leq n \leq 5$.
- The head wave amplitudes can be inverted to generate (i) the head coefficient, (ii) the ratio of the densities, and (iii) the P-wave modulus.
- The recommended number of iterations for tomographic inversion is less than 10 and commonly approximately 5.
- Ray coverage images which demonstrate large and unrealistic depths of investigation are usually the result of low resolution tomograms.
The GRM can be applied in TWO modes.

The first is the TRADITIONAL approach, which uncritically accepts the traveltime data.

For completeness, this approach should always include the hyperbolic velocity model as well as the uniform velocity model.

The second is the OPTIMIZED approach, in which the optimum XY value is used to compute an average vertical velocity.

Mt Bulga ore body case study shows that the GRM average vertical velocity best separates the shear zone (stations 45-49) from the ore body (49-53).
Velocity Models in the Sub-Weathering

- Models should also be generated for a range of XY values, in order to explicitly address epistemic uncertainty in the sub-weathering.

- The figure uses XY values of 3.75 m ± 3.75 m, for the Mt Bulga massive sulphide orebody.

- Note that the shear zone at stations 61-67 does not vary greatly with the XY value, whereas the differentiation of the shear zone (stations 45-49) from the ore body (stations 49-53) does depend on the XY value.

- Generally, the optimum XY ± half station spacing, is adequate.
The scaled density attribute, computed from the seismic velocities and the head coefficient using the head wave amplitudes, clearly differentiates the ore body (49-53) from the shear zone (stations 45-49).

The inclusion of the density in the P-wave modulus demonstrates that the low seismic velocity in the massive sulphide ore body is not a useful measure of rock strength.
Mt Bulga Ore Body – Over-Processing

- The misfit errors exhibit minimal significant change after about ten iterations.
- The figure demonstrates that the resolution is significantly reduced after fifty iterations, while the tomograms generated after one and two hundred iterations are virtually identical to those generated with the smooth vertical velocity gradient starting model.
- Over-processing has converted the vertical structures to horizontal layering.
With increasing numbers of iterations, the ray coverage demonstrates increased depth of penetration, but at the expense of decreased spatial resolution.

The ray coverage for the default smooth vertical velocity gradient tomogram has many similarities with that for two hundred iterations.
The massive sulphide ore body provides a crucial test of the lateral resolution of refraction tomography, because it is a narrow vertical feature with quite distinctive petrophysical properties.

The GRM is able to generate good estimates of the lateral extent and various useful petrophysical and geotechnical attributes, and to differentiate the ore body from the associated shear zones.

By contrast, the ore body and shear zones remain undetected, undefined and undifferentiated with the smooth velocity gradient tomogram.
Default vs GRM Tomography

- Low resolution default tomography does not detect features of major geotechnical significance, such as the 50 m wide low velocity shear zone at Mt Bulga or the 10 m low velocity ore body at Mt Bulga.

- It can be concluded that refraction tomograms generated from low resolution default starting models are not able to provide a reliable measure of the occurrence or otherwise of important or even major lateral variations in seismic velocities.

- Therefore, inversion algorithms, which generate detailed starting models, such as those of the GRM, are essential for the majority of routine geotechnical investigations.
Refraction Traveltime Inversion

- Inversion algorithms are separate from tomographic inversion.
- Inversion algorithms define the model, i.e., they tell you what you don’t know.
- Tomographic inversion refines the model, i.e., it rarely tells you what you don’t know already.
- The final tomogram is usually very similar to the starting model. Tomography rarely, if ever, improves resolution.
- Tomography is largely a smoothing or a cosmetic process.
- Tomographic inversion is inherently non-unique, that is, a multiplicity of tomograms can be consistent with the data.
- A simplistic comparison of misfit errors does not “prove” that any one tomogram is “correct,” “defensible,” or geologically reasonable.